Lecture 26 : Energy in thermal processes I

Last lecture we saw that it is possible to build a thermometer based on the melting and boiling points of water and the dependence of the volume (mercury thermometer) or pressure (gas thermometer) on temperature. Temperature is a measure of the molecular motion in the material, as we saw from the kinetic theory of gases. We also defined, $n = N/N_A$, where n is the number of moles, N is the number of particles and N_A is Avogadro's number.

Altough increasing temperature leads to expansion in most materials, there are very important exceptions, with the most important exception being water, which actually contracts (becomes more dense) as it is heated just above its melting point. In fact the maximum density of water occurs at about 4^0C. The fact that water is denser at 4^0C than ice protects the fish in lakes during winter. If ice were heaver than water, the ice would sink to the bottom of the lakes and the water exposed to the surface would freeze. As it is the ice forms a thermal barrier and enables the water at the bottom of the lake to remain unfrozen.

For an ideal gas, measurements indicate that the pressure, volume, temperature and number of moles of gas are simply related through,

$$PV = nRT = Nk_BT. \tag{1}$$

The kinetic energy of ideal gases enables us to understand this relation from simple mechanics. By considering collisions of particles with the walls of a container, we found that,

$$P = \frac{N}{3V}m\overline{v^2} \tag{2}$$

so that the pressure is related to the average speed of the particles making up the gas. From this equation we deduced that the average (translational) kinetic energy of an atom or molecule in an ideal gas is given by,

$$\overline{KE} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT \tag{3}$$

and that the root mean square speed of an atom of mass m in an equilibrium ideal gas at temperature T is given by,

$$v_{rms} = (\frac{3k_BT}{m})^{1/2} \tag{4}$$

Now we need to have a more general understanding of energy and energy transfer in materials as a function of temperature. First we define the internal energy of a material to be U. The *internal energy* U of a material is the sum of the kinetic energies (translational, rotational, vibrational etc) and potential energies of the atoms and/or molecules which make up the material. For the ideal gas we just calculated the translational kinetic energy, while for more complex materials the calculation of the internal energy is much harder and often needs to be carried out numerically. The second definition we use is the definition of heat Q, which is defined to be the energy transfer due to differences in temperature between two bodies.

The experimental observation is that every substance requires a certain amount of heat in order to raise its temperature. If no phase change occurs, we have,

$$Q = mC\Delta T \tag{5}$$

so that to raise a piece of material of mass m by a temperature ΔT, we must add an amount of energy (heat) Q. C is called the specific heat of the material and it has units J/kgK, for example the specific heat of water is $4186J/kgK$. In order to change the phase of a material, we also need to add or take away heat. The energy (heat) required to change the phase of material is called latent heat, L, and it has units of J/kg. Phase change occurs at fixed temperature and the latent heat varies from one phase change to another, for example the latent heat of fusion (melting) $L_f(ice \rightarrow water) = 3.33 * 10^5 J/kg$, while the latent heat of vaporization of water is, $L_v(water- > steam) = 2.26 * 10^6 J/kg$.

Example How long does it take for a 1kg cubic block of ice to melt on a hot summers day. Assume that the only energy absorbed is due to the sun, and that the sun provides a power of $power = 1kW/m^2$.
Solution The volume occupied by 1kg of ice is approximately found by using the relation $\rho = M/V$, and the density of ice is $920kg/m^3$. The volume is then $V = 1/920m^3$. The area exposed to the sun is then $A = V^{2/3} = .011m^2$. The energy absorbed by the iceblock due to the power from the sun is then $E = power * A * time = 11t$, where t is the time in seconds. The amount of energy required is the latent heat of fusion of water $L_f(water) = 3.33 * 10^5$. Equating the two expressions gives, $t = 30,000s$. Is this consistent with your experience. What do you think is missing??

Example

While talking on the phone, a person runs a bath which contains 50 gallons of water at $T = 65^0 C$. Realizing that this is way too hot for the baby, the person wants to cool the water down. How many liters of water at $32^0 F$ must be added to the bath to bring the temperature of the bath to the recommended value $37^0 C$? What volume of ice would be required to produce the same effect?

Solution

The volume of water, 50 gallons, is equal to

$$V = 50 * 3.786 liters = 50 * 3.786 * 10^{-3} m^3 = .1893 m^3 \tag{6}$$

The density of water is $1 g/cc = 1000 kg/m^3$, so the mass of this water is $m = 189.3 kg$. Let's define the volume of the added water to be V_a. The heat taken from the bath water goes to heating the cold water, so we have $Q_a + Q_b = 0$, so that,

$$mC(95 - 37) = m_a C(37 - 0) \tag{7}$$

Solving for m_a, we find that $m_a = m * 28/37 = 143 kg$, which is a volume of 38 gallons!! Now consider adding ice instead of cold water. In that case, melting the ice extracts an additional amount of energy from the hot water, so that,

$$mC28 = m_a C37 + m_a (3.33 * 10^5) \tag{8}$$

which leads to $m_a = 45 kg$.

The analysis of the heat absorbed by a material as a function of temperature is called *calorimetry* and it tells us a lot about the material. In a calorimeter, heat is added at a constant rate to a piece of material, whose weight is known, and the temperature is monitored.